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IEEE Guide for the Statistical Analysis of Electrical Insulation Voltage Endurance Data

Sponsor

Statistical Technical Committee of the IEEE Dielectrics and Electrical Insulation Society

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Foreword

(This Foreword is not a part of ANSI/IEEE Std 930-1987, IEEE Guide for the Statistical Analysis of Electrical Insulation Voltage Endurance Data.)

This document is the first version of a guide to promote standardized methods for the statistical analysis of data from voltage endurance tests on electrical insulation. Simple methods to calculate confidence intervals have received special attention. We strongly encourage users of this guide to calculate confidence intervals for their test data, as this will help interested parties in evaluating the precision of the tests. Much work remains to be done in extrapolating the results of tests from accelerated conditions to normal operating stress (Section 6).

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IEEE Guide for the Statistical Analysis of Electrical Insulation Voltage Endurance Data

1. Scope and References

1.1 Scope

This guide describes, with examples, statistical methods to analyze the data for time-to-failure from *constant-stress* voltage endurance tests or breakdown voltage from *progressive-stress* tests on specimens or systems of electrical insulation. Methods to compare test data are also given. The methods are principally applied to data from tests on solid insulation, but may also apply to the analyses of data from tests on gas, liquid, and composite systems. The statistical methods discussed here do not take into consideration the physical mechanism of voltage aging. The methods assume that the only aging stress is alternating voltage of constant frequency. The methods may not apply if there is more than one aging stress. Methods to ascertain the short time withstand voltage or operating voltage of an insulation system are not presented in this guide. The mathematical techniques contained in this guide may not directly apply to the estimation of equipment life.

1.2 References

This guide shall be used in conjunction with the following publications:

[1] IEEE Std 101-1972 (R 1980) , IEEE Guide for the Statistical Analysis of Thermal Life Test Data.[1]

[2] BROOKES, A. S. The Weibull Distribution: Effect of Length and Conductor Size of Test Cables. *Electra* no 33, p 49.

[3] COCHRAN, W. G. and SNEDECOR, G. W. *Statistical Methods*. Iowa State, 1976, 6th ed.

[4] DAKIN, T. W. and STUDNIARZ, S. A. The Voltage Endurance of Cast Epoxy Resins. 1978 *IEEE International Symposium on Electrical Insulation*. Philadelphia, PA: p 216.

[5] DISSADO, L. A., FOTHERGILL, J. C., HILL, R. M., and WOLFE, S. V. Weibull Statistics in Dielectric Breakdown, Theoretical Basis, Applications, and Implications. *IEEE Transactions,* EI-19, June 1984, p 227.

[6] ENDICOTT, H. S. and WEBER, K. H. Area Effect and its Extremal Basis for the Electric Breakdown of Transformer Oil. *AIEE Transactions* pt III, vol 75, June 1956, p 371.

[1] IEEE publications are available from IEEE Service Center, 445 Hoes Lane, PO Box 1331, Piscataway, NJ 08855-1331.

[7] FURTIG, K. W., MANN, N. R., and SCHEURER, E. W. Confidence Bounds and a New Goodness-of-Fit Test for the Two-Parameter Weibull or Extreme-Value Distributions with Tables for Censored Samples of Size 3(1)25. *Aerospace Research Laboratories Report,* ARL 71-0077, Wright-Patterson AFB, 1971.

[8] GLADSTEIN, D., NELSON, W., and SCHMEE, J. Confidence Limits for Parameters of a Normal Distribution from Singly Censored Samples Using Maximum Likelihood. *Technometrics,* May 1985.

[9] HAHN, G. J. and NELSON, W. A Comparison of Methods for Analyzing Censored Life Data to Estimate Relationships Between Stress and Product Life. *IEEE Transactions on Reliability,* R-23, April 1974, p 2.

[10] KIERSZTYN, S. E. Formal Theoretical Foundation of Electrical Aging of Dielectrics. *IEEE Transactions* (*Power Apparatus and Systems*), PAS-100, Nov 1981, p 4333.

[11] LAWLESS, J. F. *Statistical Models and Methods for Lifetime Data*. New York: John Wiley and Sons, 1982.

[12] LAWLESS, J. F. and STONE, G. C. The Application of Weibull Statistics to Insulation Aging Tests. *IEEE Transactions,* EI-14, Oct 1979, p 233.

[13] NATRELLA, M. G. Experimental Statistics. *NBS Handbook,* 91, 1966, pp 2–14.

[14] NELSON, W. *Applied Life Data Analysis*. New York: John Wiley and Sons, 1982.

[15] ROSEN, H., STONE, G. C. Some Graphical Techniques for Estimating Weibull Confidence Intervals *IEEE Transactions on Reliability,* R-33, Dec 1984, p 362.

[16] SIMONI, L. *Voltage Endurance of Electrical Insulation*. Bologna: Technoprint Publ, Dec 1974.

2. Introduction

2.1 Steps Required to Analyze Voltage-Aging Data

2.1.1

Select a probability distribution that adequately represents the test data. Data on solid insulation tests are most often represented by the (two-parameter) Weibull distribution [5][2] whereas the Gumbel (smallest extreme-value) distribution is often employed for liquids [6]. The lognormal distribution may also be relevant. The adequacy of the assumed distribution should always be tested, since another distribution could possibly fit better, unless experience or theory suggests a particular distribution.

2.1.2

Estimate the distribution parameters and their confidence intervals. The confidence interval objectively quantifies the uncertainty in a parameter estimate. Generally the more data, the narrower are these confidence intervals.

2.1.3

Perform a hypothesis test if data are to be compared. A hypothesis test is an objective method of ascertaining if the probability distributions of two data sets differ by a convincing amount, or whether such a conclusion is possible due to the inherent variability of the test data.

[2] The numbers in brackets correspond to those of the references in 1.2.

2.1.4

Estimate the life distribution at normal operating stress by extrapolating the accelerated aging test data. This is particularly prone to error and should be done with care, as the mathematical model, test conditions, etc, may be in error. Any conclusions derived from statistical analysis of test data are rigorously valid only for the test conditions and particular specimens used. In general, voltage endurance data are best used to make comparisons between different groups of specimens at the accelerated test levels.

3. Data Analysis

This section presents the Weibull and Gumbel probability distributions. These are the distributions most often employed to represent breakdown times, or breakdown voltages of insulation, or both. The lognormal probability distribution is also described since it is sometimes used in data analysis, especially in connection with failure models (Section 6). Also described are some of the graphical methods for statistical analysis.

3.1 Extreme-Value Distributions

The Weibull and Gumbel distributions are two types of extreme-value distributions. The extreme-value distributions have been found to represent a wide variety of failure data [2], [11], and [14].

Furthermore, the effect of the size of the test specimens (that is, thickness, area, or volume, or a combination of these) on life or breakdown voltage can easily be modeled with these distributions [2] and [14].

Statistical techniques commonly used for estimating normal distribution quantities do not apply to the extreme-value distributions. In particular, the estimates of the mean standard deviation, and the confidence intervals and hypothesis tests for the normal distribution do not apply to extreme-value distributions.

3.1.1 The Weibull Distribution

The Weibull distribution is most often used to represent both the times-to-breakdown (obtained from a constant-stress test) and the breakdown voltages (obtained from a progressive-stress test) in tests on solid insulation. The Weibull cumulative distribution function for the population fraction below *x* is

$$F(x) = 1 - e^{-(x/\alpha)^{\beta}}, x \geq 0 \tag{1}$$

where

α	= scale parameter and is positive
β	= shape parameter and is positive
x	= random variable, usually the time to breakdown or the breakdown voltage
$F(x)$	= probability of failure at voltage or time (less than or equal to) x

The probability of failure $F(x)$ is zero at $x = 0$. The probability of failure rises continuously as x increases. As the time or voltage increases, the probability of failure approaches certainty, that is, $F(\infty) = 1$.

The scale parameter α represents the time (or voltage) for which the failure probability is 0.632. It is analogous to the mean of the normal distribution.[3] The units of α are the same as x, that is, voltage, electric stress, or time.

[3] Note that a mean and a standard deviation can be determined for the Weibull distribution. These quantities, however, are seldom used.

The shape parameter β is a measure of the spread of the failure times or voltages. The larger β is, the smaller is the range of breakdown voltages or times.

The two-parameter Weibull distribution of Eq 1 is a special case of the three-parameter Weibull distribution that has the cumulative distribution function

$$\begin{aligned} F(x) &= 1 - e^{-[((x\ \gamma)/\alpha)]^{\beta}},\ x \geq \gamma \\ &= 0,\ x < \gamma \end{aligned} \tag{2}$$

The additional term γ is called the location parameter. $F(x) = 0$ for $x < \gamma$, that is, the probability of failure for $x < \gamma$ is zero. The value of γ is usually assumed to be zero in practical applications.

3.1.2 The Gumbel Distribution

The Gumbel distribution is most often used to represent the breakdown voltages of liquid and solid insulations [6]. The cumulative Gumbel (smallest extreme-value) distribution function for the population fraction below y is

$$G(y) = 1 - e^{-e^{(y-u)/b}},\ -\infty \leq y \leq \infty \tag{3}$$

where

u	= location parameter and may have any value
b	= scale parameter and is positive
y	= random variable, usually the breakdown voltage
$G(y)$	= probability of failure at voltage or time $\leq y$

The Gumbel distribution is asymmetrical and can have a physically impossible finite probability of breakdown for $y < 0$. This distribution is also called the smallest extreme-value (that is, weakest-link) distribution. If y is voltage, then the units of u and b are also voltage.

The Gumbel distribution is closely related to the Weibull distribution. That is, if x has a Weibull distribution then $y = \ln(x)$ has a Gumbel distribution

where

$$\begin{aligned} u &= \ln(\alpha) \\ b &= 1/\beta \end{aligned} \tag{4}$$

Estimation techniques (see Section 4) for one distribution (Gumbel or Weibull) apply to the other if the transformation equation, Eq 4, is utilized.

3.2 The Lognormal Distribution

The lognormal distribution has sometimes been used to represent failure data from insulation systems, but it has not been used nearly as often as the extreme-value distributions in 3.1. However, since this probability distribution is a simple logarithmic transformation of the well-known normal or Gaussian distribution, methods for data analysis are available in all standard statistical references. The probability density function of the lognormal distribution is

$$f(z) = (1/\sqrt{2\pi}\sigma)e^{[-(z-u)^2/2\sigma^2]} \tag{5}$$

where

z = log x
x = breakdown voltage or time-to-failure
μ = logarithmic mean
σ = logarithmic standard deviation

The cumulative density function is the integral of the above. There is no closed-form equation for the integral. Values of the distribution are found in [3], [13], or can be obtained from statistical calculators or computer programs.

3.3 Censored Data

Censoring of data occurs when n specimens are started on the test together and the failures of only r ($< n$) are observed.

Censoring is encountered mainly with constant stress tests, where the data are analyzed or the test is terminated before all the specimens fail. Censoring can also occur with progressive stress tests where flashovers or spurious breakdowns occur on specimens that would otherwise survive to high stresses. Censoring can occur by plan or by accident in many insulation tests, and this factor must be taken into account in the data analysis.

There are also other kinds of censoring, such as progressive and multiple censoring. These types of censoring result when failed specimens are disqualified because of specimen failure by spurious mechanisms. Analytical treatment of such data is difficult. See [11] and [14].

3.4 Probability Graph Papers

Special graph papers for the Weibull and Gumbel distributions are shown in Figs 1, 2, 4, and 5 respectively. One axis of the graph paper is a nonlinear cumulative probability scale (This scale is the same for the Weibull and the Gumbel papers). The other axis is the data axis; logarithmic for the Weibull distribution and linear for the Gumbel distribution. This axis is for the failure times or voltages. The axes are scaled so that plotted data from the two-parameter Weibull and Gumbel distributions tend to follow a straight line. Such graph paper is commercially available.[4]

Similar graph paper is also available for the lognormal distribution. The probability scale is the same as for normal distribution graph paper. The other axis however, is logarithmic. If lognormal paper is not available, normal paper can be employed by taking logarithms of the data and plotting them on the linear scale.

The primary uses of the probability plotting paper are to assess the adequacy of the assumed distribution to represent the test data and to obtain rough parameter estimates. Linearity on probability paper is the only easy method to assess which of the Weibull, Gumbel, or lognormal distributions best represents the data. All test data should first be plotted to check that the assumed distribution is adequate.

3.4.1 Weibull Probability Paper

Samples of Weibull probability paper are shown in Figs 1 and 2. To use this paper, order the failure times or voltages from smallest to largest. The most commonly used approximation of the cumulative probability F_i is

$$F_i = \left(\frac{i}{n+1}\right) \cdot 100\% \qquad (6)$$

The Weibull example data in Table 1 are plotted in Figs 1 and 2. Data from unfailed specimens should not be plotted, although they should be included in n.

[4] TEAM, Box 25, Tamworth, NH 03886.

Table 1—Breakdown Data on Epoxy Specimens

Specimen No i	F_i %	Breakdown Time t_i (h)
1	10	15.3
2	20	30.3
3	30	48.5
4	40	89.4
5	50	90.4
6	60	105.7
7	70	144.9
8	80	—
9	90	—

If a reasonably straight line is produced by the plotted data, then it may be reasonable to assume that the data are adequately represented by the Weibull distribution. Some random deviations from a straight line are normally expected. If, however, there is a consistent departure from a straight line (for example, curvature), then another distribution may fit the data better. Figure 3 shows plots on Weibull probability paper of other distribution functions.

3.4.2 Gumbel Probability Paper

A sample of Gumbel probability paper is shown in Figs 4 and 5. The n failure voltages (or times) are ordered from smallest to largest, and the i^{th} smallest voltage y_i can be assigned the plotting position

$$G_i = \left(\frac{i}{n+1}\right) \cdot 100\% \tag{7}$$

The Gumbel data in Table 2 are plotted in Figs 4 and 5. If a reasonably straight line is produced by the plotted data on the Gumbel paper, then the Gumbel distribution adequately represents the failure data. Examples of data from other distributions are plotted on Gumbel paper in Fig 6.

3.4.3 Lognormal Probability Paper

A sample of lognormal probability paper is shown in Fig 7, and contains the data presented in Table 3. The failure data are ordered from smallest to largest, and the i^{th} smallest voltage x_i can be assigned the plotting position

$$F_i = \left(\frac{i}{n+1}\right) \cdot 100\% \tag{8}$$

If a reasonably straight line is produced by the plotted data on the lognormal paper, then this distribution adequately represents the data.

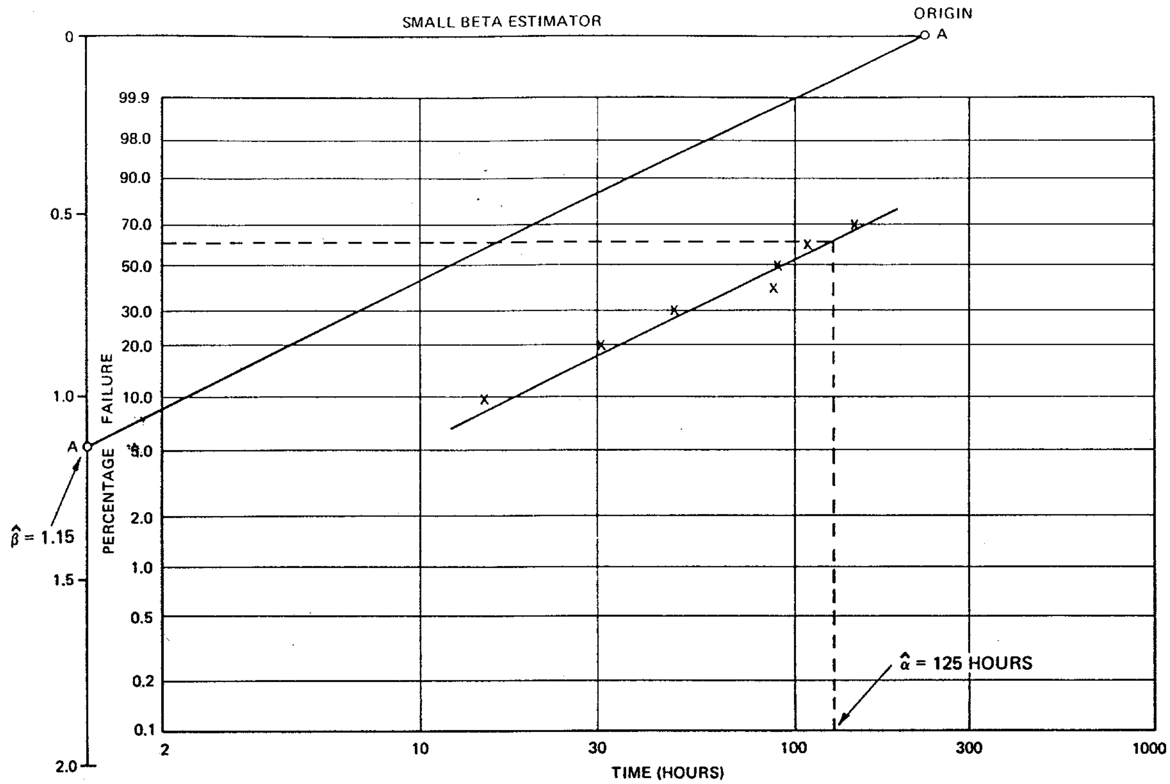

Figure 1—Weibull Plot of Failure Data With Line Fit By Eye

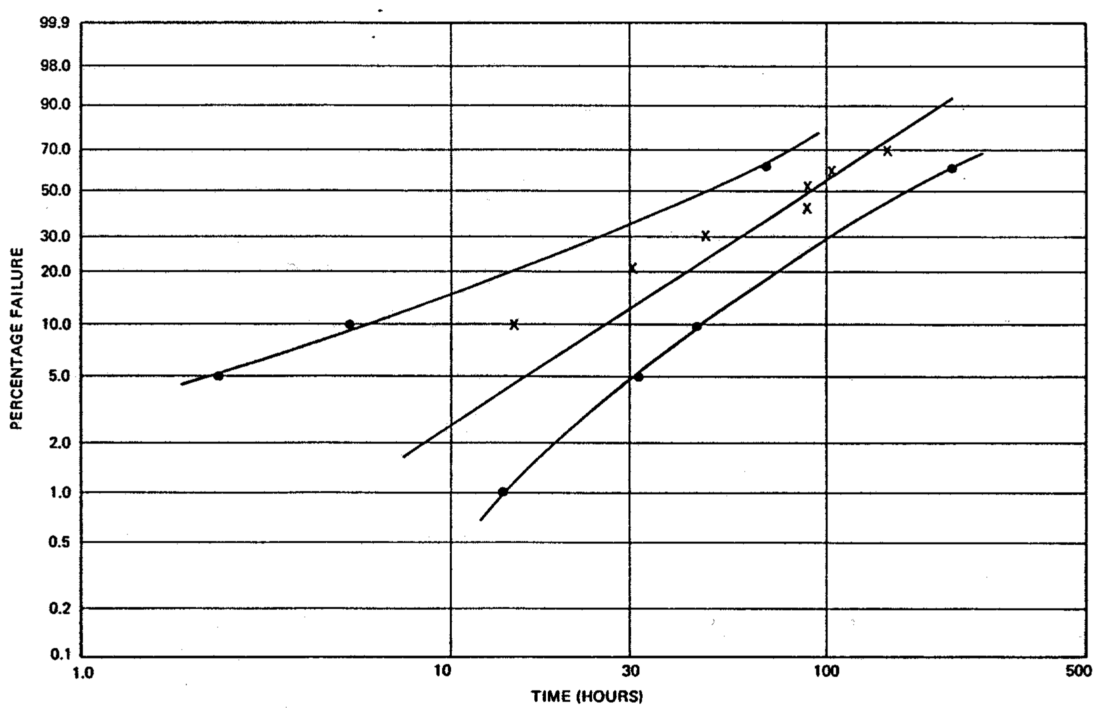

Figure 2—Weibull Plot of 90% Confidence Bounds for Percentiles and Maximum Likelihood Fitted Line

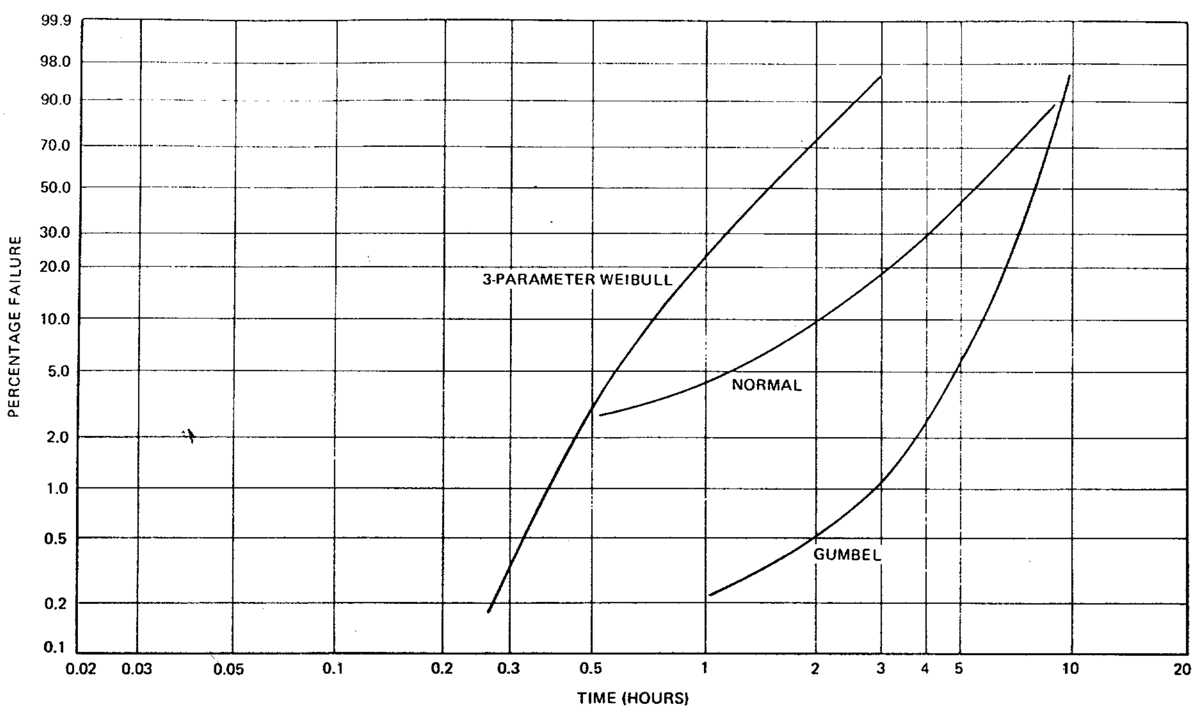

Figure 3—Plots of Other Distributions on Two-Parameter Weibull Paper

Table 2—Breakdown Voltage of Oil

Specimen No i	G_i %	Breakdown Voltage y_i (kV)
1	9	5.0
2	18	5.0
3	27	5.2
4	36	5.6
5	45	5.7
6	54	5.7
7	64	5.8
8	73	5.8
9	82	—
10	91	—

An example of the Weibull distribution function plotted on lognormal paper is shown in Fig 8.

4. Estimation of Distribution Parameters

4.1 Graphical Parameter Estimates

The probability graph papers described in 3.4 can be used to obtain approximate estimates of the parameters. The slope and intercept of a straight line fitted *by eye* to the data yield parameter estimates. More objective estimates can be calculated by fitting a straight line to the plotted data using least-squares regression. Parameter estimates based on a least-squares analysis are not as accurate as other estimates, and are therefore not preferred. There are no graphical confidence intervals for these estimated parameters.

4.1.1 Weibull Graphical Estimates

Plot the test data on Weibull probability paper, as described in 3.4.1. Fit a straight line *by eye* to the data points. The estimate for the scale parameter α, denoted by $\hat{\alpha}$ is the time (or voltage) corresponding to $F(x)$ = 63.2%. An estimate for the shape parameter, denoted as $\hat{\beta}$, is obtained as follows. The line labelled A-A through a point labeled *origin* in Fig 1, is drawn parallel to the fitted line. The point where this line interesects the shape scale is an estimate for β. In the example, the estimate is $\hat{\beta} = 1.15$.

4.1.2 Gumbel Graphical Estimates

Plot the test data on Gumbel probability paper, as described in 3.4.2. Fit a straight line *by eye* to the data points. The estimate of the location parameter u, denoted by $\hat{u}$, is the voltage corresponding to $G(y)$ = 63.2%. An estimate of the scale parameter b, denoted by $\hat{b}$, is the difference in y between the 63% and 31% points.

For the data in Table 2, plotted in Fig 4, $\hat{u}$ = 5.84 kV. Using the voltages corresponding to the 63rd and 31st percentiles, $\hat{b}$ = 5.84 kV - 5.43 kV = 0.41 kV.

4.1.3 Lognormal Graphical Estimates

Plot the test data on lognormal probability paper, as described in 3.4.3. Fit a straight line *by eye* to the data points. An estimate of the log mean, denoted by $\hat{z}$, is the voltage (or time) corresponding to $F(z) = 50\%$. An estimate of the log standard deviation, denoted by s, is the difference in z (= log x) between the 16% and 50% points. Note that $\bar{z}$ is dimensionless, though it can be converted back to time or voltage by taking the antilog.

For the data in Table 3, plotted in Fig 7, $\bar{z} = 1.113$ (x = 13 h) and $s = 0.16$.

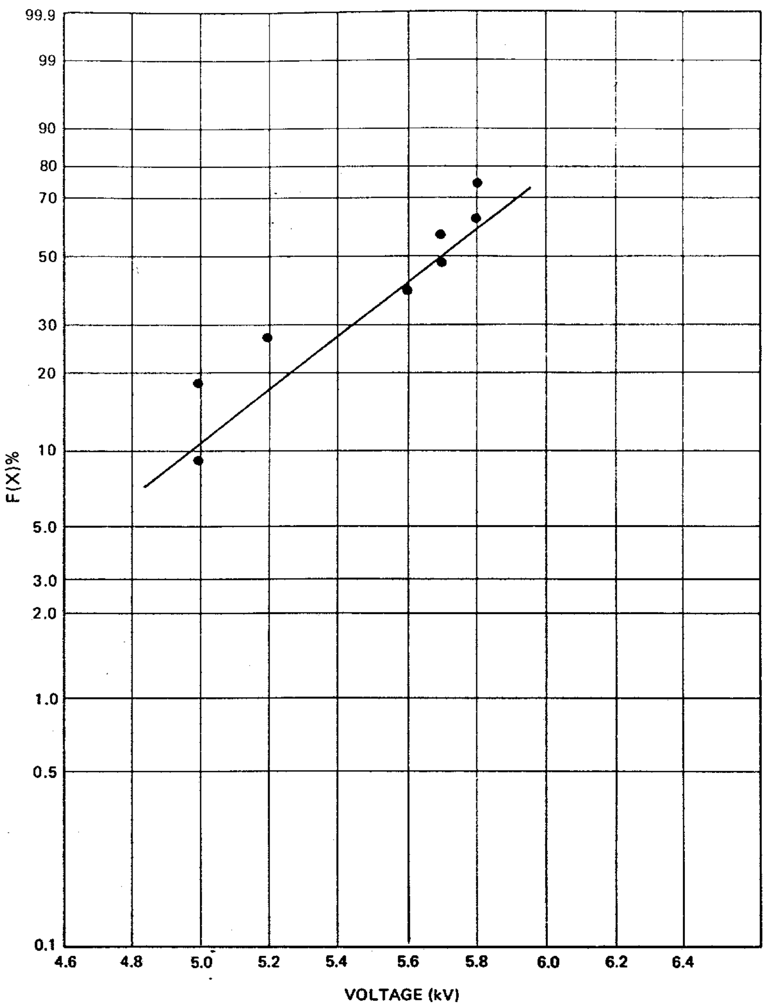

Figure 4—Gumbel Probability Plot of Data In Table 2 With The Line Fitted By Eye

Table 3—Breakdown Data

Specimen No i	F_i %	Breakdown Time x_i (h)	Log (time) z_i
1	10	7.0	0.84
2	20	8.5	0.93
3	30	11	1.04
4	40	12	1.08
5	50	12	1.08
6	60	17	1.23
7	70	18	1.25
8	80	18	1.25
9	90	21	1.32

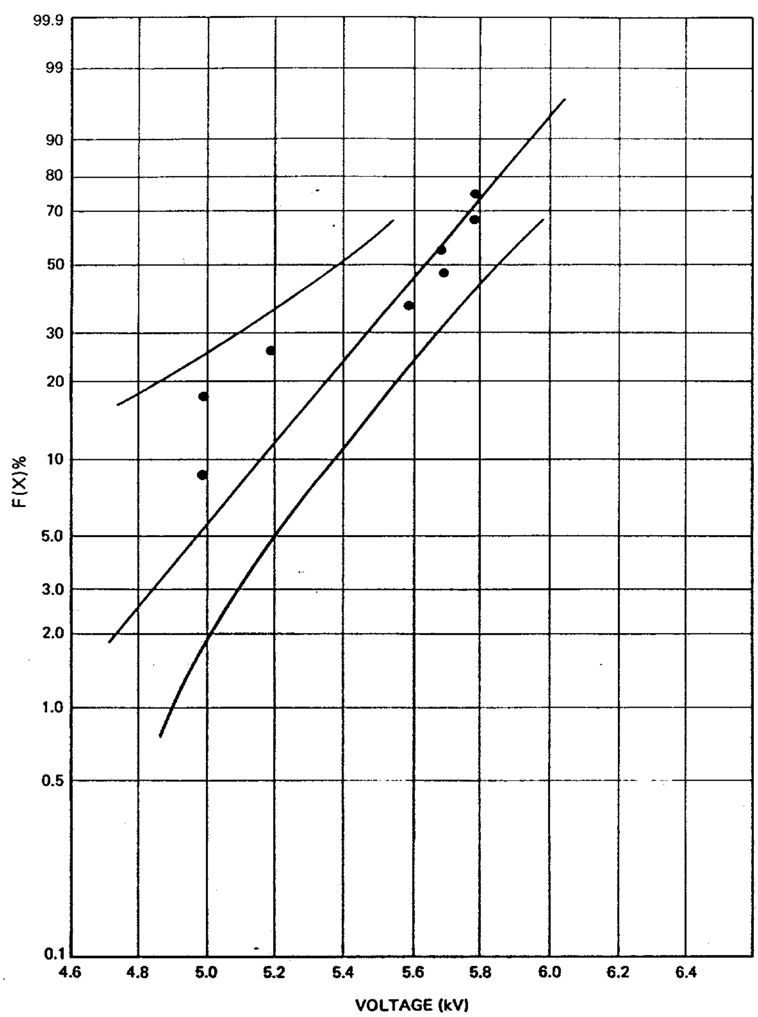

Figure 5—Gumbel Plot of 90% Confidence Bounds on Maximum Likelihood Fitted Line

4.2 Accurate Parameter Estimates

Graphical estimates of the distribution parameters are not the most accurate. There are, however, more accurate methods, which also yield confidence intervals. For the extreme-value distributions, the most widely employed parameter estimates are the maximum likelihood estimates [11] and [14]. The parameter estimates for the lognormal distribution are based on the well-known formulae used for the normal distribution.

4.2.1 Weibull Distribution

For the two-parameter Weibull distribution, the maximum likelihood estimate of β from singly censored data is the iterative solution of the maximum likelihood equations (see Appendix A). A typical short BASIC program for solving these equations on a personal computer is shown in Appendix A. This program requires an initial approximate value for β. This is most easily obtained from a Weibull probability plot. Otherwise, an initial β of 10 may be used for progressive-stress tests. For constant-stress tests, an initial β of 2 may apply.

The maximum likelihood estimates for the data in Table 1 are $\hat{\alpha} = 115$ h and $\hat{\beta} = 1.5$ h. The line on Weibull probability paper that fits this data is shown in Fig 2. The line is plotted from two points obtained by substituting two values of x, along with $\hat{\alpha}$ and $\hat{\beta}$ into 1 and calculating the corresponding values of $F(x)$.

4.2.2 Gumbel Distribution

There are equations for the maximum likelihood estimates for the Gumbel distribution. However, it is simpler to transform the failure data and then use a computer program that fits the Weibull distribution. In the Gumbel data y_i are transformed into Weibull data with

$$x_i = e^{y_i} \qquad (9)$$

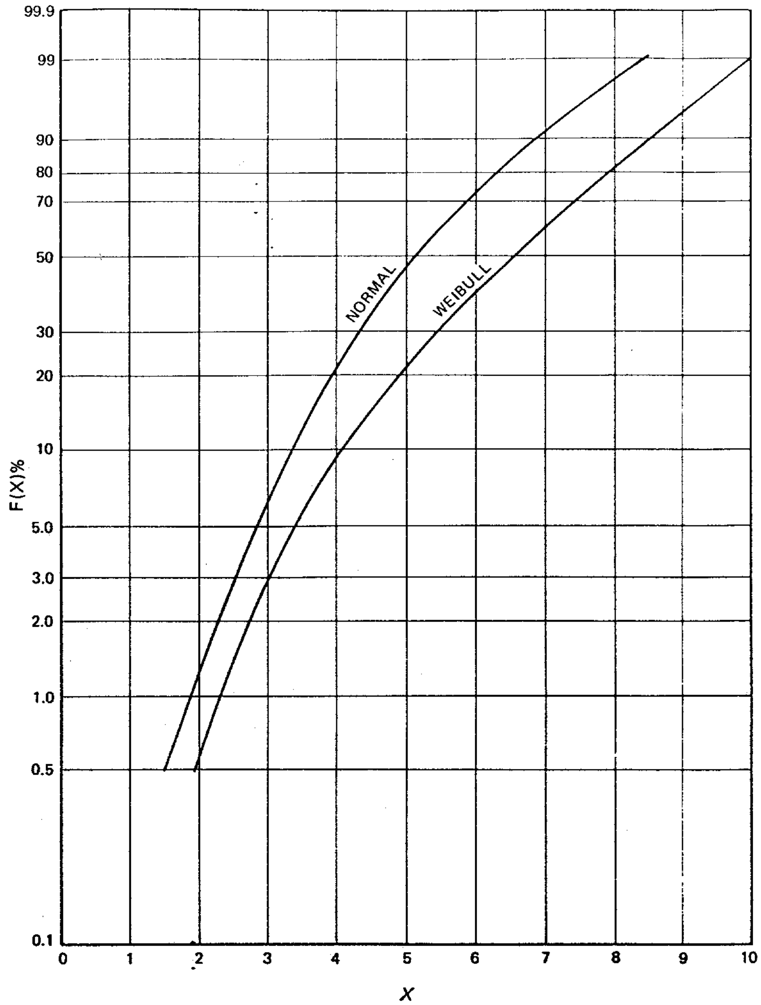

Figure 6—Plots of Other Distributions on Gumbel Probability Paper

An initial estimate for $\beta = 1/b$ can be obtained from a Gumbel probability plot. Alternatively, assume an initial b of 0.1 (that is, $\beta = 10$) or $0.5(\beta = 2)$. The program output of β and α are then converted to Gumbel parameter estimates by using Eq 4.

For the Gumbel data in Table 2, $\hat{u} = 5.73$ kV and $\hat{b} = 0.26$ kV. With $\hat{b}$ and $\hat{u}$ and two sets of y values corresponding values of $G(y)$ can be calculated. These two points define the straight line on Gumbel paper (Fig 5).

4.2.3 Lognormal Distribution

Exact estimates for the lognormal parameters are available if there is no censoring, that is, $r = n$. These estimates are obtained by taking the logarithms of the failure voltages (or times) and using the transformed data with well-known formulae for the mean and standard deviation of the normal distribution. Using the notation in Eq 5.

$$\bar{z} = \frac{\Sigma z}{n} \qquad s = \sqrt{\frac{\Sigma(z-\bar{z})^2}{(n-1)}} \tag{10}$$

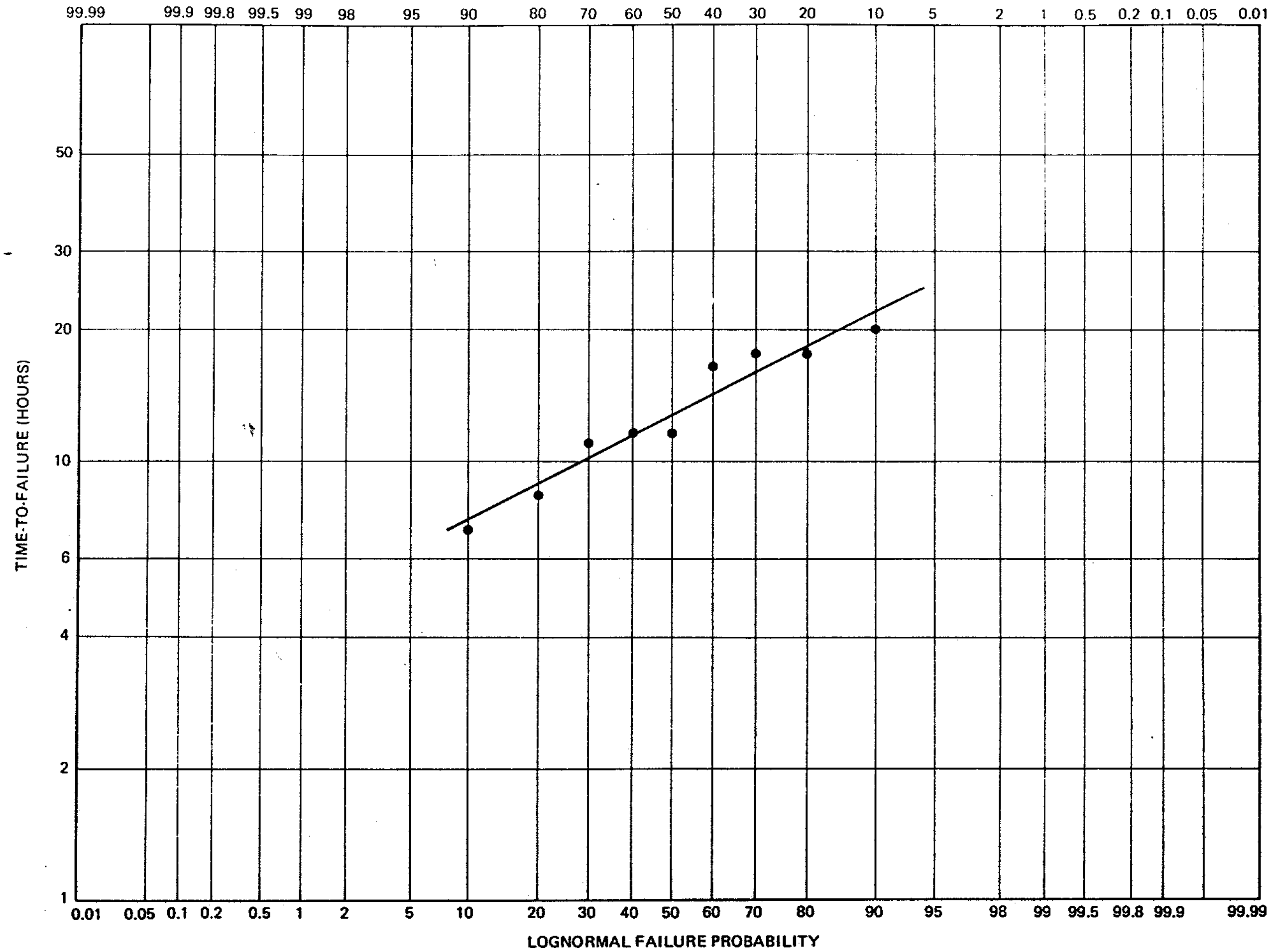

Figure 7—Lognormal Probability Plot of Data In Table 3 With The Line Fitted By Eye

The quantities are easily calculated with statistical calculators or computer software, which is widely available. If the data are censored, then the special methods described in [13] are required.

4.3 Confidence Intervals for Parameters

If the same experiment involving the testing of many specimens is performed a number of times, the values of the parameter estimates $\hat{\alpha}$, $\hat{\beta}$; $\hat{u}$, $\hat{b}$ or $\bar{x}$, s from each experiment differ. This variation in estimates results from the statistical nature of insulation breakdown. Therefore, any parameter estimate differs from the *true* parameter value that is obtained from an experiment involving an infinitely large number of specimens. Hence, it is common to give with each parameter estimate a *confidence interval* that encloses the true parameter value with high probability. In general, the more specimens tested, the narrower the confidence interval. Enough specimens should be tested so as to obtain sufficiently narrow confidence intervals for practical purposes. If the confidence intervals are calculated to be adequate before all the samples have failed, the test can be aborted.

If an experiment is poorly performed, for example, if the applied voltage is not held constant in a constant stress test, the confidence intervals are inaccurate. Confidence intervals are valid only for *identically* tested specimens.

4.3.1 Approximate Weibull Parameter Intervals

There are various methods of estimating confidence intervals for Weibull parameters [11] and [14]. One method, which is valid for censored data and all sample sizes, requires a computer program that is not generally available [12]. Many other computer programs give parameter estimates and approximate confidence intervals. Some of these are outlined in references [11] and [14]. Some of the methods used in these programs may not be accurate, especially with small sample sizes. If a particular program yields results within approximately 10% of those shown in Appendix B, then the computation method is probably valid for the given sample size. In addition to computer programs, confidence-limit tables have been calculated for the extreme-value distributions [7], [11], and [14].

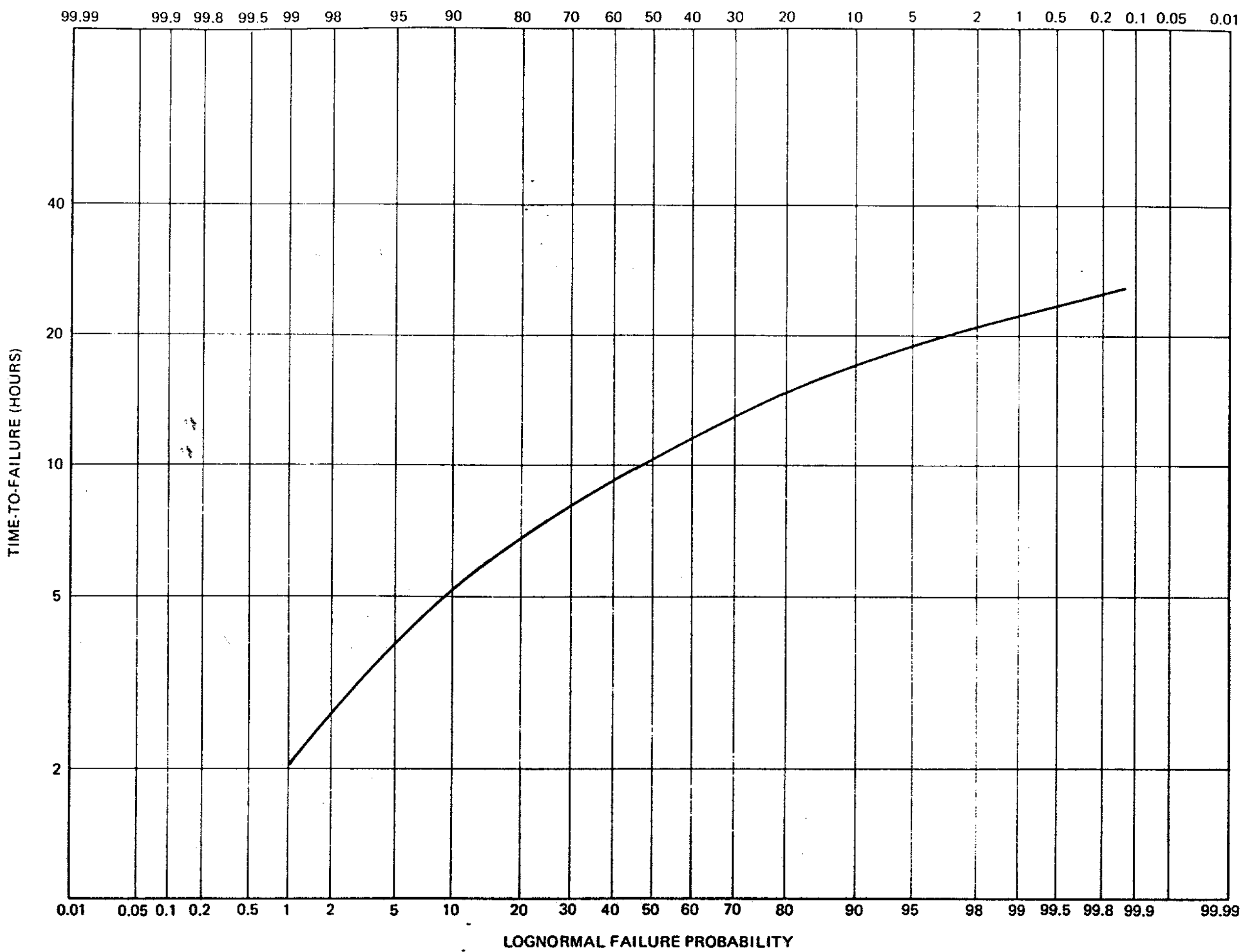

Figure 8—Plot of a Weibull Function on Lognormal Probability Paper

The confidence interval tables for α and β, which cover all possible censoring fractions are very extensive [7]. For simplicity these tables are represented as curves in Figs 9, 10, and 11. These curves are approximate and are for experiments when up to 25 specimens are tested. Figures 9, 10, and 11 give the 90% confidence intervals only [15]. For more accurate intervals, other sample sizes, confidence limits other than 90%, etc, a computer-based method or reference to Tables is necessary [7], [11], [12], and [14].

Figure 9 is used to obtain factors W_l and W_u for the two-sided 90% confidence limits for the shape parameter β

$$\beta_l = W_l\hat{\beta}$$
$$\beta_u = W_u\hat{\beta} \qquad (11)$$

where

β_l and β_u= lower and upper limits, respectively, for the interval

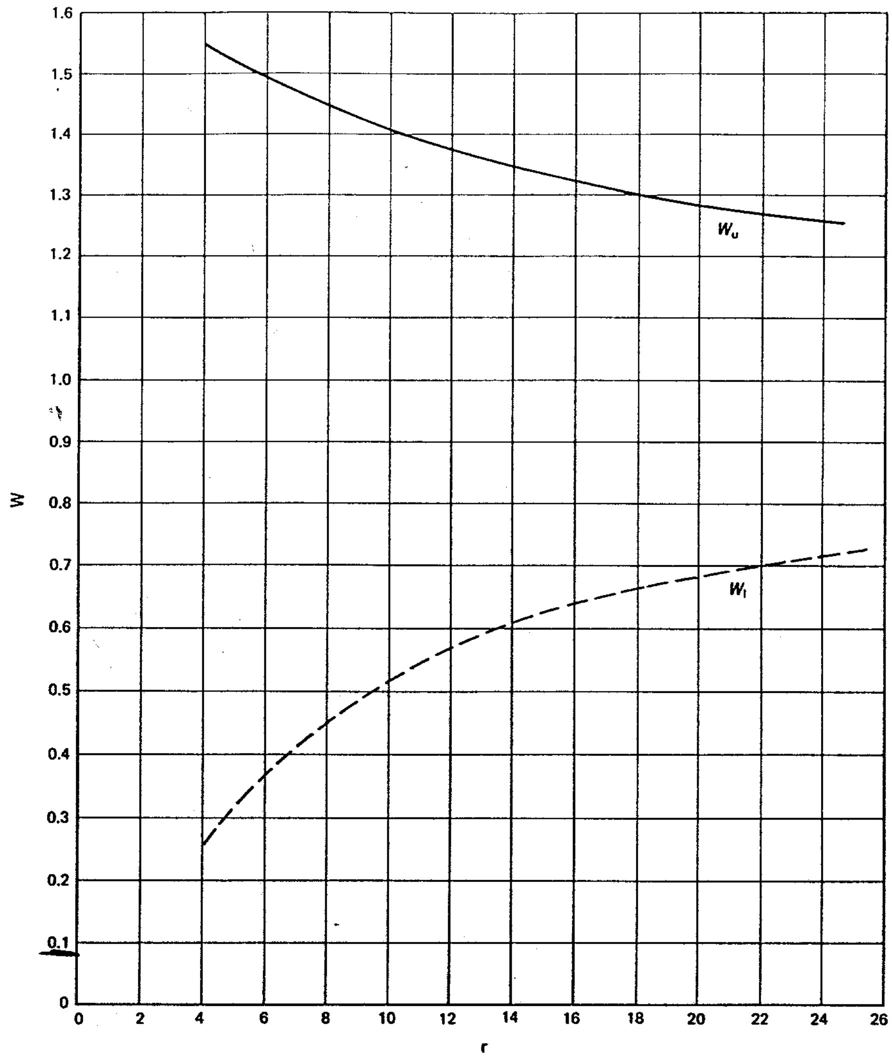

Figure 9—W Factors for Calculating 90% Confidence Intervals for β and b

As seen in Fig 9, W_l and W_u are mainly functions of r, the number of failures. For the Weibull data in Table 1, W_l = 0.42 and W_u = 1.46, and since $\hat{\beta}$ = 1.5 (4.2.1), the 90% confidence limits for β are β_l = 0.42 X 1.5 = 0.63 and β_u = 1.46 X 1.5 = 2.19.

Figures 10 and 11 are used to calculate the 90% confidence interval for α

$$\alpha_1 = \hat{\alpha}^{-Z_1/\hat{\beta}}$$

$$\alpha_u = \hat{\alpha}^{-Z_u/\hat{\beta}} \tag{12}$$

where

α_l and α_u= lower and upper confidence limits, respectively

The factor Z_l (Fig 10), is primarily a function of n, the number of specimens put on test. However, Z_u (Fig 11), is a function of both r, the number of specimens that actually failed, and n. For the Weibull data in Table 1, Z_l = 0.75 and Z_u = 1.0. Therefore, the 90% confidence limits for α are α_l = 69 and α_u = 220 h.

4.3.2 Approximate Gumbel Parameter Intervals

The curves for the Weibull parameter intervals apply to the Gumbel parameters. The curves in Figs 9, 10, and 11 yield approximate 90% confidence intervals.

The approximate 90% confidence interval for b has lower and upper limits

$$b_1 = \hat{b}/W_u$$

$$b_u = \hat{b}/W_1 \tag{13}$$

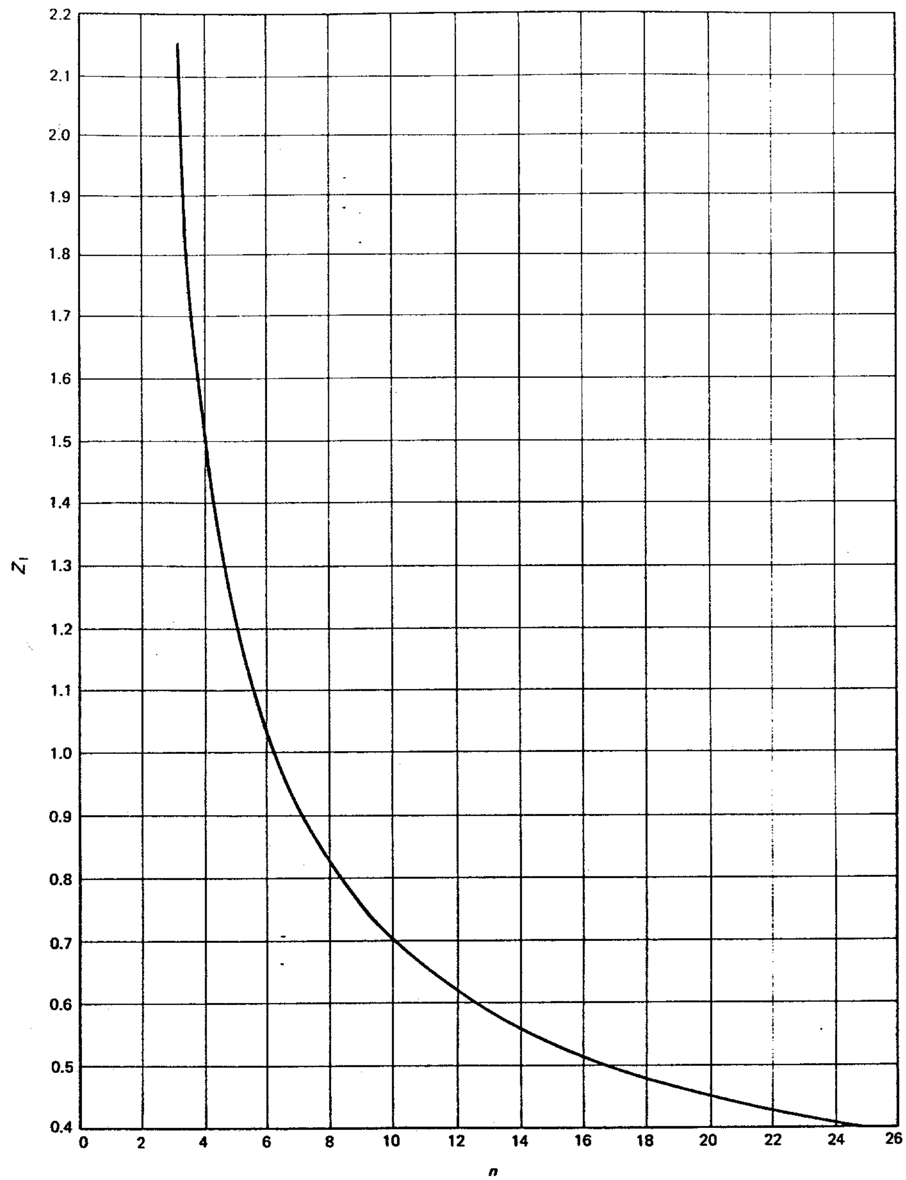

Figure 10—Z_1 Factor to Calculate 90% Confidence Interval for α_1 and u_1

Note that W_u factor from Fig 9 is for the lower bound and W_l is for the upper bound. For the Gumbel data in Table 2

$W_u = 1.44$
$W_l = 0.45$

thus

$b_l = 0.18$ kV
$b_u = 0.58$ kV

are the 90% confidence limits for b.

The approximate 90% confidence interval for u has lower and upper limits

$$u_l = \hat{u} - \hat{b}Z_l$$
$$u_u = \hat{u} + \hat{b}Z_u \quad (14)$$

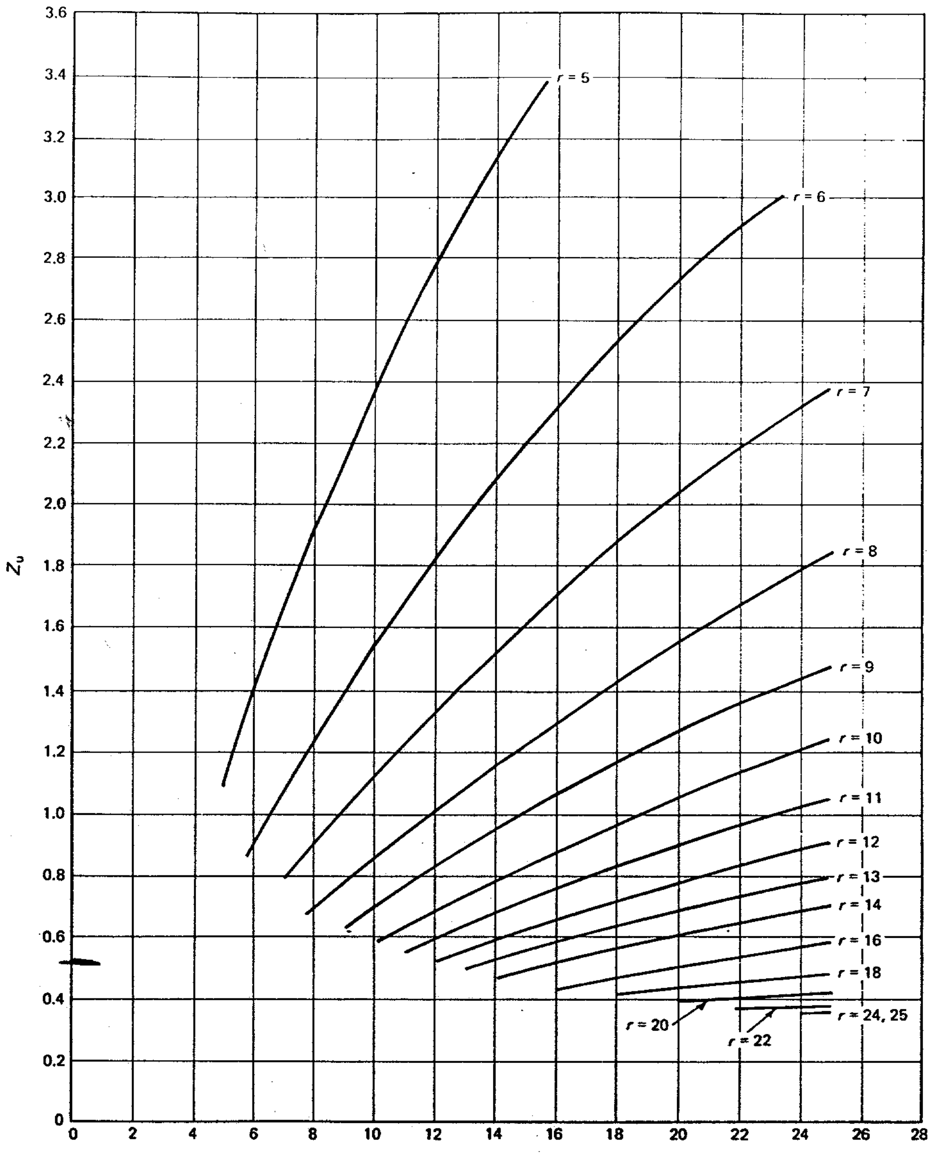

Figure 11—Z_u Factor to Calculate 90% Confidence Bounds for α_u and u_u

The Z factors are obtained from Figs 10 and 11. For the Gumbel data in Table 2, $Z_l = 0.70$ and $Z_u = 1.12$. Thus the 90% confidence interval for u has limits $u_l = 5.57$ kV and $u_u = 6.04$ kV.

4.3.3 Lognormal Parameter Intervals

The confidence intervals for the log mean and log standard deviation are easily found using the student's t and X^2 distributions, if there is no censoring. The 90% confidence interval for z has the lower and upper limits

$$z_1 = \bar{z}\frac{t_{0.05}*s}{\sqrt{n}}$$

$$z_1 = \bar{z} + \frac{t_{0.05}*s}{\sqrt{n}} \qquad (15)$$

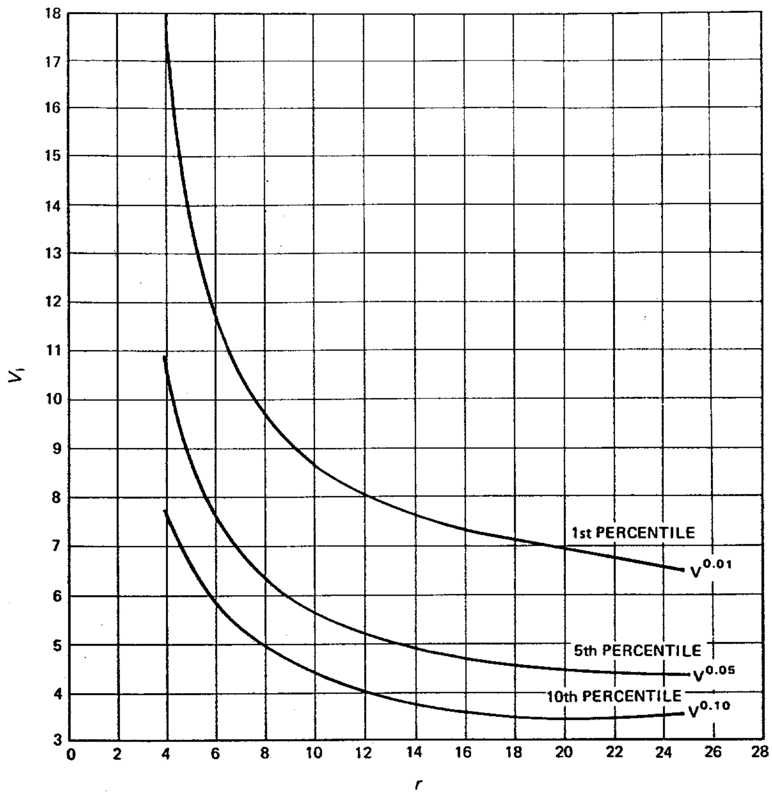

Figure 12—V_1 Factor for Calculating the 90% Tolerance Bounds for Gumbel and Weibull 100 p^{th} Percentiles

where $t_{0.05}$ is obtained from a student's t table, with n-1 degrees of freedom. Such a table is found in any standard statistics textbook [3]. The lower and upper limits for the 90% confidence interval for the log standard deviation are

$$s_1 = \sqrt{\frac{(n-1)s^2}{\chi_{0.05}^2}} \qquad (16)$$

$$s_u = \sqrt{\frac{(n-1)s^2}{\chi_{0.95}^2}} \qquad (17)$$

where X^2 is from the X^2 tables, with n-1 degrees of freedom. Such tables are in any standard statistics textbook [3]. Confidence intervals for the parameters when only censored data are available, can be estimated from the methods described in [8].

4.4 Confidence Intervals for Percentage Failed

Often it is of interest to estimate the time to failure or the failure voltage at a low probability of failure. The values of *x* or *y* at a particular probability of failure are called percentiles. Confidence intervals for percentiles should also be calculated. Although not often used, percentiles and their confidence intervals for the lognormal distribution are estimated by the methods described in [8] and [13].

Many Weibull computer programs give estimates and confidence intervals (sometimes known as tolerance bounds) for percentiles. Since some programs use confidence limits or approximations that are not valid for data with few failures, programs should be checked for accuracy with the example data in Appendix B.

Approximate methods are available to calculate confidence intervals for percentiles, with the same theoretical basis as the intervals in 4.3, for those without access to a valid computer program. Tables upon which the curves in Figs 12 and 13 are based are only available for the 1st, 5th, and 10th percentiles, and 90% confidence intervals.

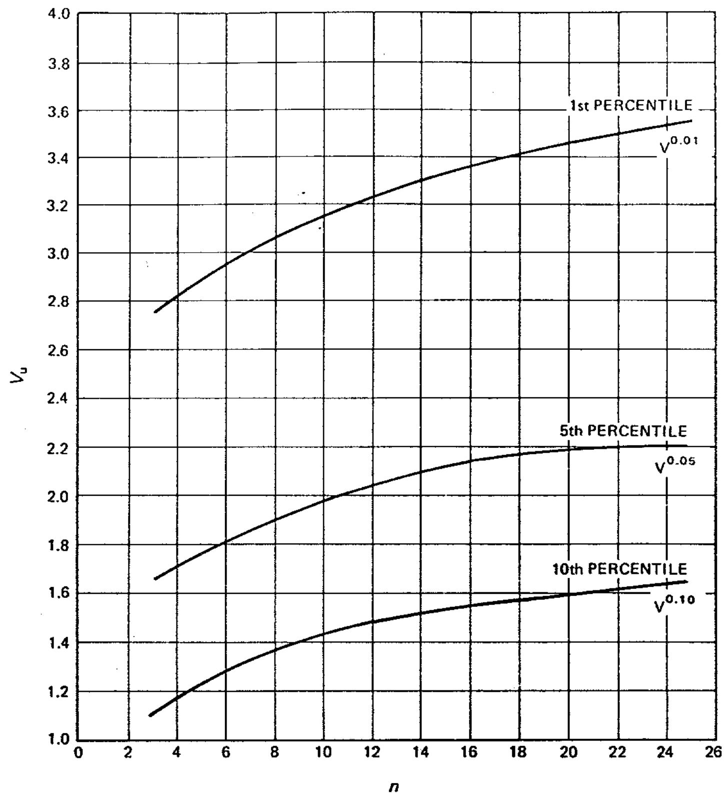

Figure 13—V_u Factor for Calculating the 90% Tolerance Bounds for Gumbel and Weibull 100 p^{th} Percentiles

4.4.1 Weibull Percentiles

The maximum likelihood estimate of the p^{th} quantile, or the 100 p^{th} percentile, for the Weibull distribution is

$$\hat{x}_p = \hat{\alpha}[-\ln(1-p)]^{1/\hat{\beta}} \tag{18}$$

where

$\hat{\alpha}$ and $\hat{\beta}$ = maximum likelihood parameter estimates

The 90% confidence interval for x_p is

$$x_1(p) = \hat{\alpha}^{[-V_1(p)/\hat{\beta}]}$$
$$x_u(p) = \hat{\alpha}^{[-V_u(p)/\hat{\beta}]} \qquad (19)$$

where

x_l (p) and x_u (p)= lower and upper limits for x(p), the 100 pth percentile, respectively

The V factors are obtained from Figs 12 and 13 for the 1st, 5th, and 10th percentiles (p = 0.01, 0.05, and 0.10 respectively). From Fig 12 V_l (p) is primarily only a function of r whereas V_u (p) from Fig 13 is primarily a function of n.

For the Weibull data in Table 1, for the first percentile, V_l (0.01) = 10.4 and V_u (0.01) = 3.1. Thus the 90% confidence limits are x_l (0.01) = 0.11 and x_u (0.01) = 14 h. The maximum likelihood estimate of the first percentile is $\hat{x}_p$ = 5.3 h.

The confidence limits for the percentiles, together with the confidence interval for α (the 63rd percentile) can be usefully displayed on Weibull probability paper. For the upper limit, plot the calculated limits (x values) corresponding to the 1st, 5th, 10th, and 63.2 (α) percentiles on the graph paper. Join these four points with a smooth line. Similarly, draw a line through the plotted lower confidence limits. Such confidence curves enclose any particular percentile of the true population with 90% probability. The greater the number of specimens tested, the closer the upper and lower curves.

4.4.2 Gumbel Percentiles

The maximum likelihood estimate of the pth quantile, or the 100 pth percentile, for the Gumbel distribution is

$$\hat{y}_p = \hat{u} + \hat{b} \ln\left[\ln\left(\frac{1}{1-p}\right)\right] \qquad (20)$$

where

$\hat{u}$ and $\hat{b}$ = maximum likelihood parameter estimates

The 90% confidence interval for y_p is

$$y_1(p) = \hat{u} - \hat{b}V_1(p)$$
$$y_u(p) = \hat{u} - \hat{b}V_u(p) \qquad (21)$$

where

y_l (p) and y_u (p)=the lower and upper limits for the 100 pth percentile. The V_l (p) and V_u (p) factors for the relevant percentile are obtained from Figs 12 and 13, respectively.

For the Gumbel data in Table 2, for the 5th percentile, V_l = 6.4 and V_u = 2.0, thus the 90% confidence limits are y_l (0.05) = 4.1 and y_u (0.05) = 5.2 kV. The maximum likelihood estimate of the 5th percentile is y_p = 4.95 kV. The confidence bounds for the percentiles, together with the confidence bounds for b, can be plotted on Gumbel paper (Fig 5).

5. Comparison Tests

A common situation involves testing two or more insulation types or groups of specimens to determine which of the two is *superior*. To analyze results from such a test involves testing the hypothesis to verify that there is no difference between the probability distributions of the data for the two types of insulation. The *hypothesis test* usually compares the parameters or a given percentile, for example, 63^{rd}, 50^{th}, or 10^{th}. For power system insulation, it is usually of more interest to compare low percentiles in ranges of one percent or lower.

It is easiest to compare test data from two types of insulation by plotting data sets on probability paper. However, visual comparison of two plots is subjective.

5.1 Rigorous Hypothesis Tests

Various methods are available to test the hypothesis to verify that the theoretical distribution of one set of test data differs *significantly* (convincingly) from another. Methods for the lognormal distribution are easily adapted from the normal distribution, using the log transformation. See [3], ch 4 for an example. Rigorous hypothesis tests for the Weibull and Gumbel distributions, described in [11] and [14], are usually difficult to perform and often require sophisticated computer programs.

5.2 Simplified Method to Compare Percentiles of Extreme Value Distributions

If two data sets differ convincingly the following method is used to give an approximate assessment. Determine if the confidence intervals for a chosen percentile of the two distributions overlap. If there is no overlap, for example, at the 10^{th} percentile, then the two 10^{th} percentiles differ significantly at the selected confidence level. This comparison does not assume that the two shape parameters are equal. The confidence intervals for the percentiles are calculated as described in Section 4.

It is always useful to compare sets of test data on Weibull probability paper. Plot the data from the two (or more) tests on the same graph paper.

Table 4—Breakdown Stresses (kV/mm) Polyethylene for Two Manufacturing Processes

Failure No	Process 1 (Unscreened)	Process 2 (Screened)
1	35	39
2	35	45
3	36	49
4	40	49
5	43	53
6	43	53
7	43	53
8	46	53
9	46	55
10	48	55
11	48	57
12	48	57
13	48	57
14	48	57
15	48	61
16	51	64
17	51	64
18	51	65
19	51	67
20	57	68

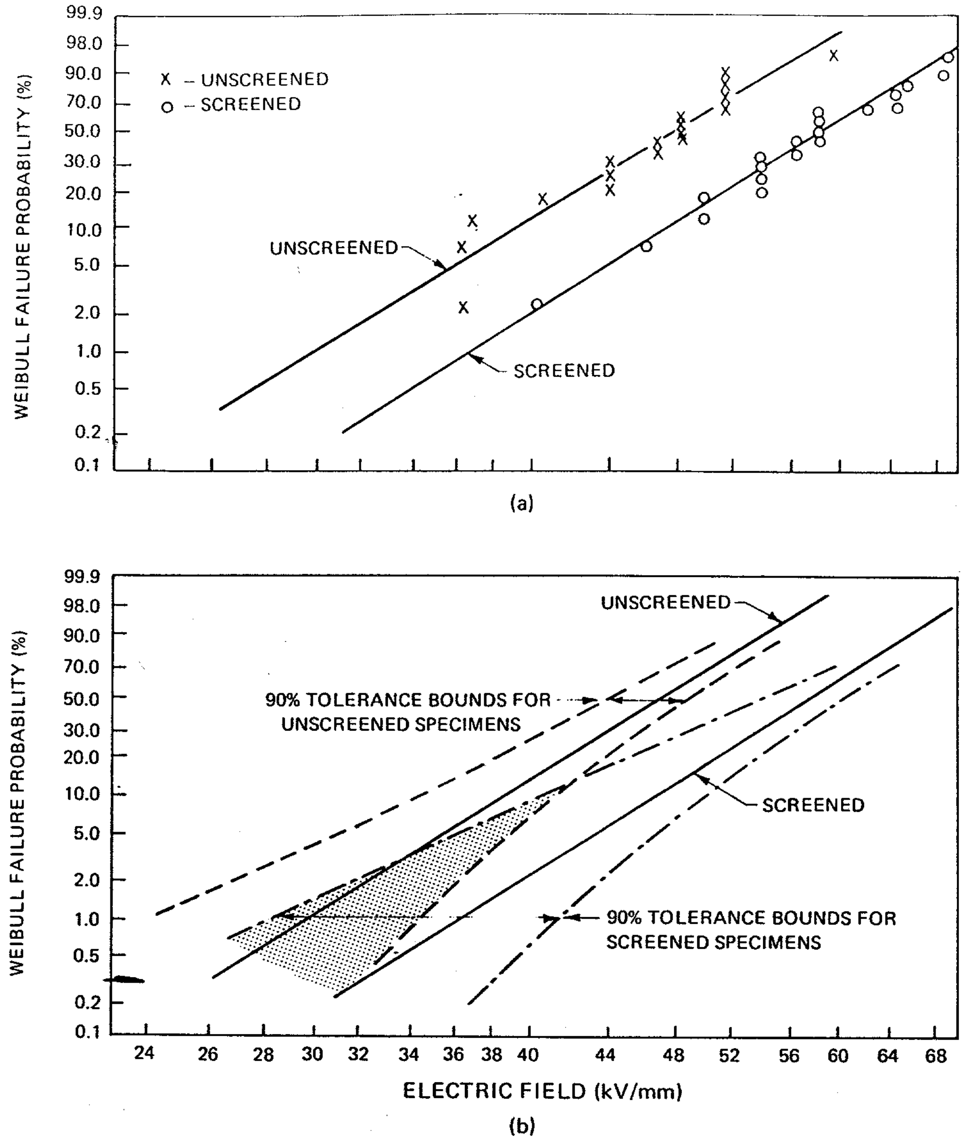

Figure 14—Comparison of Data Sets

As described in 4.4, plot the 90% confidence bounds for percentiles for each data set. The test data in [12] and Table 4 are plotted in Fig 14 with the 90% confidence intervals. For percentiles above approximately 10%, the two intervals do not overlap. Therefore, high percentiles of the two insulations differ significantly. Note that for low percentiles, the intervals overlap, and those percentiles probably do not differ significantly. In principle, more specimens need to be tested to show if low percentiles differ significantly.

6. Failure Models

A failure model may result in an equation that relates the test stress to the time-to-failure. In general as the test voltage decreases, the failure times increase. The general experimental procedure is to test several specimens at two or more voltages. Since the times-to-failure at a constant stress are variable, finding the best failure model is often difficult.

Two major classes of failure models have been suggested; the inverse power model and the exponential model. Examples of their use can be found in [4], [9], [10], and [16]. No model has been conclusively proven to be valid, thus caution is required in employing the results of an analysis.

6.1 Inverse Power Model

The inverse power model is described by

$$L = kV^{-N} \tag{22}$$

where

L	= time-to-failure at voltage V
V	= voltage
k	= constant
N	= constant

Usually L is the Weibull scale parameter α, the mean or some other failure percentile. As the voltage increases, the life decreases. The inverse power model is a good description if the data (the life at each test voltage) are plotted on log-log graph paper, and a straight line results (Fig 15).

6.2 Exponential Model

One form of the exponential model is described by

$$L = c\mathrm{e}^{-kV} \tag{23}$$

where

L	= time-to-failure at voltage V
V	= voltage
c	= constant
k	= constant

The exponential model is a good description if the failure data plot as a straight line on semilog paper, with time-to-failure the logarithmic scale, and voltage the linear scale.

Other variations on Eq 23 have been suggested [4] and [8]. Equations 22 and 23 can result in very large differences in predicted times-to-failure when extrapolated to low-voltage stresses.

6.3 Fitting Data to Failure Models

Objective methods for estimating the parameters of a failure model are based upon transforming the equation into linear form, and employing linear regression (curve-fitting). Since simple linear regression requires the times-to-failure at any particular voltage to be a symmetric distribution, such a regression on failure data that follow an extreme-value distribution is not strictly valid. Various approximate procedures, generally valid for tests with many samples, are discussed in [9] and [14], ch 12.

An alternative approach is to assume the failure data are lognormally distributed. The logarithms of the failure times are then normally distributed, and thus simple linear regression may be applied. This approach is the basis of IEEE Std 101-1972 (R 1980) [1], where the Arrhenius equation is the failure model for thermal aging.

The open circles are the means (based on the lognormal) for the time-to-failure at each test voltage.

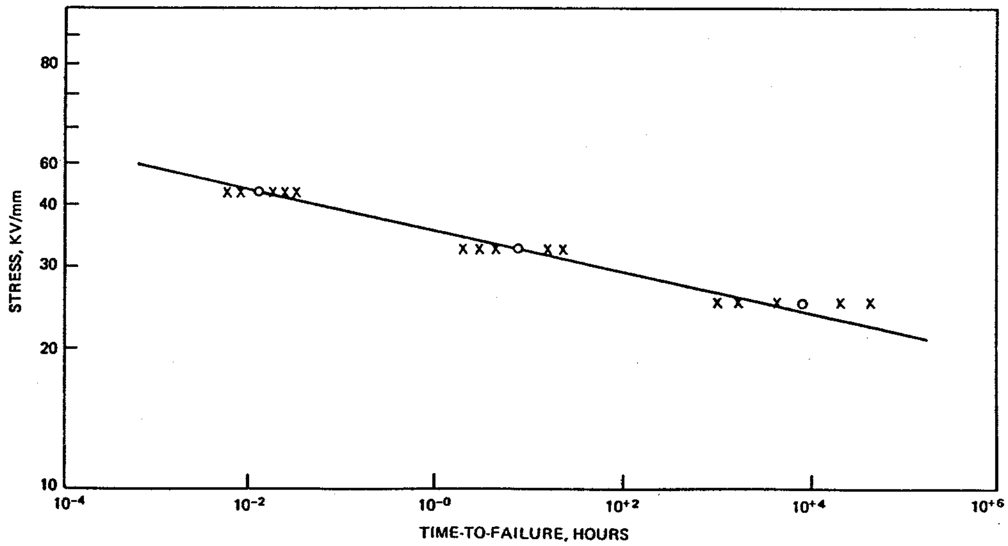

Figure 15—Inverse Power Model

The parameters *k* and log *c* are then estimated in exactly the same way as for Eq 24.

6.3.1 Inverse Power Model

The linearized form of Eq 22 is

$$\log L = \log k - N \log V \tag{24}$$

This equation has the linear form

where

y	$= mx + b$
y	$= \log L$
x	$= \log V$
m	$= -N$
b	$= \log k$

The parameters are to be estimated by linear regression. The assumption is that at any particular test voltage *V*, log *L* is normally distributed.

Computer software for estimating log *k* and *N* are widely available. Such software often yields confidence intervals for the equation parameters. Estimates of the life at any particular voltage with confidence intervals, can also be obtained. Such estimates assume that the failure mechanism is unchanged at the selected voltage.

6.3.2 Exponential Model

The linearized form of the simple exponential model of Eq 23 is

$$\ln L = \ln c - kV. \tag{25}$$

Annex A

(Informative)

Program to Estimate Weibull and Gumbel Parameters

(These Appendixes are not a part of ANSI/IEEE Std 930-1987, IEEE Guide for the Statistical Analysis of Electrical Insulation Voltage Endurance Data, but are included for information only.)

The maximum likelihood parameter estimates for the two-parameter Weibull distribution require the iterative solution of

$$\frac{A_2}{A_1} - \frac{1}{\hat{\beta}} - C = 0$$

and

$$\hat{\alpha} = \left(\frac{A_1}{r}\right)^{1/\hat{\beta}}$$

where

$$A_k = \sum_{i=1}^{r} x_i^{\hat{\beta}} (\ln x_i)^{k-1} + (n-r) x_s^{\hat{\beta}} (\ln x_s)^{k-1}$$

$$k = 1, 2, 3$$

$$C = \frac{1}{r} \sum_{i=1}^{r} \ln x_i$$

x_s = the highest common unfailed running time or test voltage
n = total number of specimens tested
r = number of specimens failed

A BASIC program for solving this equation, based on a Newton-Raphson routine, is shown in Fig A-1. A_3 is required by the program. Note that the logarithms in the program are to the base e.


```
10 REM  THIS PROGRAM CALCULATES THE LIKELY MAXIMUM
12 REM  ESTIMATES OF WEIBULL PARAMETERS
15 REM  A FIRST GUESS FOR BETA IS REQUIRED
18 REM
20 DIM TIME(25),LNTIME(25),A(3)
30 PRINT"INPUT N-NO. OF SAMPLES TESTED"
40 INPUT N
50 PRINT"INPUT R-NO. OF SAMPLES FAILED"
60 INPUT R
70 PRINT"INPUT TS-TIME OR VOLTAGE TEST STOPPED"
80 INPUT TS
85 LTS=LOG(TS)
90 PRINT"INPUT FIRST ESTIMATE FOR BETA"
100 INPUT BETA
110 ITER=0
120 C=0
130 PRINT"INPUT THE FAILURE TIMES OR VOLTAGES"
140 FOR I=1 TO R
150 INPUT TIME(I)
160 LNTIME(I)=LOG(TIME(I))
170 C=C+LNTIME(I)
180 NEXT I
190 C=C/R
200 FOR J=1 TO 3
210 SUM=0
220 FOR K=1 TO R
230 SUM=SUM+((TIME(K)^BETA)*(LNTIME(K)^(J-1)))
240 NEXT K
250 A(J)=SUM+(N-R)*(TS^BETA)*(LTS^(J-1))
260 NEXT J
270 QUOT=A(2)/A(1)
280 FPRIME=A(3)/A(1)-(QUOT^2)+((1/BETA)^2)
290 F=QUOT-(1/BETA)-C
300 BETA=BETA-F/FPRIME
310 ITER=ITER+1
320 IF ITER>20 THEN 370
330 IF ABS(F)>0.0001 THEN 200
340 ALPHA=(A(1)/R)^(1/BETA)
350 PRINT"ALPHA=";ALPHA
352 PRINT"BETA =";BETA
354 PRINT"ITERATIONS=";ITER
360 STOP
370 PRINT"DID NOT CONVERGE"
380 PRINT"NEED A BETTER ESTIMATE OF BETA"
390 STOP
400 END
```

Figure A-1—Basic Program to Calculate $\hat{\alpha}$ and $\hat{\beta}$

Annex B

(Informative)

Examples

The following examples are the *exact* 90% confidence bounds for the given data [11]. Valid computation methods should yield similar results. The Weibull distribution is assumed.

Example 1:

Failure Times (minutes) 11, 22, 33, 60, 100, 133, 151, 159, 205, 319, 340, 378, 605, 629, 1060, 1132, 1279

n = 20
r = 17
$\hat{\alpha}$ = 575 min
$\hat{\beta}$ = 0.77
β_l = 0.50
β_u = 1.00
α_l = 355
α_u = 1089

1st percentile:

t_l = 0.05
t_u = 6.7

5th percentile:

t_l = 1.3
t_u = 35

10th percentile:

t_l = 5.4
t_u = 75

Example 2:

Failure Times (minutes) 140, 160, 185, 280, 290, 295, 320, 320

n = 14
r = 8
$\hat{\alpha}$ = 342 min
$\hat{\beta}$ = 3.8
β_l = 1.7
β_u = 5.5
α_l = 299
α_u = 473

1st percentile:

t_l = 26.4
t_u = 149

5th percentile:

t_l = 67.8
t_u = 203

10th percentile:

t_l = 102
t_u = 234